

The Legacy of Mario Pieri

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Elena Anne Marchisotto, James T. Smith, *The Legacy of Mario Pieri in Geometry and Arithmetic*. Birkhäuser Boston, Basel, Berlin, ISBN 13: 978-0-8176-3210-6

Mario Pieri (1860-1913) has until recently been a neglected figure in the history of geometry in Italy. Various papers by Elena Marchisotto have kept his name alive, as also have the paper by Avellone, Brigaglia, and Zappulla and papers by Bottazzini. Now we have this book, the first fruit of a collaboration between Marchisotto and James Smith. The result is the best chance the mathematical community has had to appreciate the contributions of this interesting figure since his death.

Pieri was geometer, drawn to the group around Peano that shared his aim of writing mathematics to a very high standard of formal rigour. To this end, Pieri even published papers in the highly symbolic formalism Peano had invented, but more importantly he worked with great energy and imagination to produce a truly rigorous system of axioms and deductions for elementary geometry. His presentation of mathematics as a hypothetico-deductive system was appreciated outside of the geometrical setting in which it was devised, and the axiomatic work of the Italians and Pieri particularly preceded Hilbert, so a further question for historians becomes to assess the impact of Pieri's work, which was well received in its day but shortly became eclipsed. As the authors discuss, there is, however, an interesting connection to the work of Tarski.

Pieri axiomatised real and complex projective geometry, inversive geometry, neutral geometry, Euclidean geometry, and arithmetic. This book looks at what Pieri did and its subsequent reception, and gives one long and one much shorter memoir of Pieri's in an English translation, but it also casts its net widely. We get short biographies of many different people, an equally careful account of places in Italy, and an exploration of the rise and fall of Pieri as well as that of Peano.

The memoirs presented here in translation are ‘Point and Sphere’, a lengthy work that dates from 1908 and presents an axiom system for elementary Euclidean geometry, and the much shorter ‘On the axioms of arithmetic’ from 1907. These show the thoroughness of Pieri’s work, and also some of its range because most scholarship on Pieri has discussed the work he did before Hilbert on projective geometry. We must wait for these to appear in English in later volumes, which will also involve other authors. The authors compare each work with what else was going on, so in the case of ‘Point and Sphere’ they provide a comparison with the work of Hilbert and Veblen, which axiomatised versions of Euclidean geometry and projective geometry respectively. The comparisons are short and helpful. Most noteworthy in this book is the careful logical analysis of Pieri’s work that is provided, which makes it very clear the price in logical complexity that Pieri paid to obtain rigour; some statements in his systems are much more complicated than in the alternative axiomatisations. To quote (p. 283) ‘Pieri achieved economy of primitive concepts, but at the cost of extreme complexity of postulates’.

The book is welcome in other ways: it is very well illustrated, it has a very thorough bibliography, and of course it carries a richly informative biography of Pieri himself. The completed series will surely restore not only Pieri but the milieu around him and Peano to a central place in the history of modern geometry.

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