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Matemáticas en el siglo XIX

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A. N. Kolmogorov, A. P. Yushkevich (eds.) *Mathematics of the Nineteenth Century. Geometry, Analytic Function Theory*. Translated from the Russian by Roger Cooke. Basel: Birkhauser Verlag, 1996. ISBN 0-8176-5048-2 and 3-7643-5048-2

This volume is part of a multi-volume work on the history of mathematics in the nineteenth century which first appeared in the Soviet Union in the early 1980s. The volume under consideration, as the title states, contains two long chapters, the first, on geometry, by B. L. Laptev and Boris A. Rozenfel'd; and the second, on analytic function theory, by A. I. Markushevich. The two essays share certain aspects of their general historiographical approach, which I will discuss first, but they are really separate entities which I will treat individually in what follows.

The aim of these chapters is to provide a relatively detailed survey of the topics which they treat. Of course, at lengths of about 115 and 150 pages respectively, they are not truly comprehensive studies, and they concentrate on mathematics that is consecrated by hindsight as being of interest. Thus there is a strong concentration on what the authors and editors considered to be the most important topics and writers from the standpoint of the third quarter of the twentieth century. The focus is very much on the mathematical content of the work, rather than on social, institutional, or even mathematical context, and the mathematics often seems to have a life of its own, existing platonically in a world where great mathematicians occasionally intervene. This reader was often reminded of the approach of other (essentially contemporary) survey works, such as Morris Kline's *Mathematical Thought from Ancient to Modern Times* or the *Abrégé d'histoire des mathématiques, 1700-1900*, edited by Jean Dieudonné. They take as their task explaining major points of the mathematical development to the interested mathematician who is already familiar with the subject. As I discuss below, a large number of more recent works may make us question the value of this self-imposed limitation of approach. However, taking the essays on their own terms, they may be praised for scrupulous and (then) up-to-date scholarship, an interesting selection of material, and a high degree of readability aided in no small measure by the very fine English and mathematical prose of Roger Cooke. Mathematicians with interests in these fields will find themselves better served than in Kline's volume, in part because of the greater detail. Historians with research interests in these areas should certainly know these essays for the overall accounts which they provide, as they —particularly the chapter of Markushevich— contain relevant insights while establishing various benchmarks for the historical writer. A strength of both chapters is their inclusion of Russian work, often neglected yet very much a part of trends in Europe in the period. The account is not narrowly nationalistic, and presents much work that deserves to be better known.

Markushevich, writing on analytic function theory, has written the more successful account. The unity of the subject matter and the close relationship between the different technical parts of the subject has allowed him a nicely connected narrative beginning with the eighteenth century roots of the subject. Elementary complex function theory is integrated with the theories of elliptic and modular functions in this treatment, allowing a view of the full richness of the subject as it was seen at the time. Indeed the author points this out explicitly, remarking that the work of Riemann, Weierstrass, Briot and Bouquet was valued by contemporaries above all for its use in the study of special classes of analytic functions, above all elliptic and Abelian functions. More space could have been accorded to why these fields were considered to be of interest, perhaps by looking at their research into their uses in solving differential equations. A considerable amount of mathematical detail is provided where necessary, especially in areas where the modern non-specialist reader may be on less familiar ground.

Helpful historical insights on technical points are often well-expressed. As an example, consider Markushevich's analysis of the 1831 Cauchy theorem guaranteeing that an analytic function of a complex variable has a convergent Maclaurin expansion and providing a geometric series which majorizes the expansion. Markushevich [p. 149] notes:

> It is crucial to emphasize that these majorants and the corresponding estimate of the remainder of a power series provide the investigator with

> much more than the mere fact of uniform convergence in each disk concentric with the circle of convergence and having radius less than the circle of convergence. In other words, with his theorem Cauchy essentially removed the question of uniform convergence for power series, proposing a simple quantitative characterization of it.

Many historians have felt a need to account for Cauchy's failure to see the requirement of uniform convergence earlier in his career, but here an historical point, based on analysis of the mathematical content of the paper, is used in a sensitive way to make clear why the concept was not relevant to Cauchy, at least in this context. Such nice insights pervade this text, making it consistently stimulating.

Many nice details of historical transmission of ideas also figure in this account, though unfortunately not always with the kind of documentation that will satisfy historians. For example, we learn [p. 237] that Weierstrass informed Kovalevskaya and H. A. Schwarz of his product expansion for entire functions in 1874, a result which was published only in 1876. However there is no indication of where this information came from. And all too often a line of mathematical development is explained as a 'natural' development. When we hear that "in the study of power series the baton passed from Cauchy to Abel" [p. 148], we would like very much to have some idea about how the baton was passed, and even what the baton was! The text nevertheless does draw our attention to changes in the textbook —presentation of the subject, for example, inviting lines of inquiry about how both the techniques and the tastes were acquired which led to the developments described.

The Geometry chapter I liked less, though it may in part be due to my greater familiarity with the material on function theory. It is also due to the rather heterogeneous nature of the material being treated and the sheer volume of material under discussion. The chapter is divided into seven parts, which (I say it again) are retrospectively defined, artificially creating divisions between certain kinds of material that were originally much closer together, as well as uniting things that were at the time quite separate. Vectors and Grassmann's Ausdehnungslehre thus receive their principal treatment under the heading of 'algebraic geometry' and 'geometric algebra' which takes up only eight pages of the total, surely an underrepresentation given the literature. Non-Euclidean and multidimensional geometry receive a more leisurely treatment —as one might expect in an article co-authored by Rozenfel'd— and the result is more satisfying. 'Analytic and Differential Geometry' is separately treated from 'Projective Geometry', under which heading we find the analytic projective geometry of Möbius and Plücker. Despite this choppy effect, there are still many good connections made between the distinct parts —so that the relation of Möbius' work to the later definition of free vectors is mentioned in the section on projective geometry, though not while vectors are under discussion.

In this chapter too I found the use of modern language more troubling. Thus Somov is said [p. 25] to have developed vector methods in the 1860s, though no details are given to indicate whether he literally anticipated Gibbs and Heaviside or whether instead we are speaking merely of some piece of mathematics in which we can recognize vectorial techniques. The values of the present are also freely visited on the past. we are told that an important 'defect' of Steiner's work was the absence of a "distinct use of the principle of signs" [p. 37]

A further result of the necessary brevity is the dropping of historically tantalizing information, once again without references: for example: "in the mid-nineteenth century there was an acrimonious controversy between the proponents of the synthetic and analytic methods in projective geometry, the two sides accusing each other of mixing projective and metric concepts" [p. 40]. We do not learn who was involved, where they worked, the relation of their point of view to their own work and that of their students, and so on. Likewise the priority dispute between Gergonne and Poncelet over the invention of duality is recounted, but it is up to the reader to find the relevant literature. Despite these reservations, the chapter is interesting and useful, explaining many rather obscure points well and providing a good introduction to the material.

In reading both of these chapters I was again and again struck by how much good work has been done in the period intervening between the Russian edition and the English one. It is really a pity that this work, which is certain to make its way into many mathematics libraries, contains no appendix with references to more recent historical studies adding to our understanding of the mathematical world of the past. Detailed studies could have been cited by Lutzen, Rowe, Gray, Scholz, Belhoste, Bottazzini, Ferreirós, and a host of others. These have greatly enriched our understanding of the mathematical world of the nineteenth century, both as regards the content and the mathematical and institutional context in which the mathematicians lived and worked. More recent historians have understood thoroughly that the mathematical world of the past formed that of the present day, rather than the reverse, and that we cannot expect the preoccupations and values of the present to inform the work of the mathematicians of a century ago.

In conclusion, though, I recommend these chapters to historically-interested readers. I do not have a copy of the original at hand to compare with the translation, though I have struggled with the Russian in the past and am very grateful to the translator for a remarkable job. The idea of presenting the work in English translation is a good one, and any serious mathematical library should contain this book and its companions —despite their dated character— for the fine job they do of presenting much beautiful mathematics in a relatively short space.

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