Abel's Proof: An Essay on the Sources and Meaning of Mathematical Unsolvability. Peter Pesic. Cambridge, MA: MIT Press, 2003. 213 pp. $24.95. ISBN 0-262-16216-4.

Reviewed by: Alejandro R. Garciadiego, Universidad Nacional Autónoma de México

Niels Henrik Abel (1802–29) is credited with solving an extremely difficult mathematical problem. His talents were unintelligible to his peers, and Abel died poor and ignored, but his significance in the history of mathematics is undeniable. At first glance it would appear from these observations that Pesic's book clearly falls beyond the chronological framework of this journal. Furthermore, other reviewers have already claimed that Pesic's rendi-

tion of the mathematics involved is not necessarily clear—see Lards Gårding, *Notices of the American Mathematical Society* 51 (2004): 332; and, even more, that in some cases, "Pesic gets something dead wrong," according to Gerald B. Folland, *American Mathematical Monthly* 111 (2004): 78. Nonetheless, there are specific reasons to recommend the reading of this monograph to early modern scholars, even if they are not mathematicians or scientists.

Indeed, Abel solved an extremely difficult problem: he showed that, if only using arithmetical operations and extractions roots, it is impossible to find a general formula for the roots of a polynomial equation of degree 5 or higher, a question that, on the one hand, goes back to the origins of written mathematics and on the other, proved to be unsolvable under the original premises. The precise historical origins of this particular question are unknown, but high school algebra courses deal with the issue when addressing the form

$$ax^2 + bx + c = 0$$

using different methods. Ancient cuneiform tablets show that the Babylonians had already developed techniques to find some of the solutions to this equation. Over time, the problem grew in complexity and importance, and mathematicians attempted to find solutions to equations of higher degrees. By the sixteenth century, some of the most prominent mathematicians—Nicolò Tartaglia (ca. 1500–57) and Hieronimo Cardano (1501–76), among others—attempted to find solutions to equations of third and fourth degrees. In fact, histories of mathematics are rich in myths and legends concerning the debates, challenges, and threats motivated just by the discussions over the priority of some of these discoveries. Even more, at this time, the problem of finding solutions to equations of higher degree acquired such relevance within mathematics that François Viète (1540–1603) claimed "to have found the master art that would solve any algebraic problem" (47). So, briefly, the main question originated (at least) with the Babylonians and was solved in the nineteenth century; but it was in the sixteenth century when it acquired its pivotal importance.

Readers of this journal will find Pesic's book useful for a number of reasons. In the first place, Pesic does not limit himself to narrating the chronological evolution of the proof itself. But, as the title of the monograph suggests, he goes much farther and discusses the significance of the roots and consequences of Abel's demonstration. For this reason, Pesic begins his book by discussing some of the difficulties the Greeks confronted when analyzing the concept of "number." This, obviously, is a fundamental issue in the history of mathematics. When discussing the Greeks, Pesic immediately argues that they used to dissect the category of quantity into two: that of number and that of magnitude, a distinction that modern mathematicians ignore and which immediately shows that the evolution of mathematics is not necessarily progressive, nor accumulative, nor lineal, and not necessarily going from the simplest to the more complex. This is extremely important because most general textbooks on the history of mathematics reflect an evolution that is clearly supportive of simpler developments. But the discussion of this very first topic reveals that there were traditions in mathematics that have been obscured and forgotten by later advancements. We should infer that we cannot interpret classical works (especially Greek) using our contemporary conceptions.

Farther on, it also becomes clear that the history of mathematics portrays some historiographical and methodological characteristics of its own. For example, at the time of the Scientific Renaissance, mathematicians not only turned to the new translations of the classical Greeks; they incorporated new methodologies to advance new knowledge. Mathematicians were eager to disclose how the Greeks had discovered their results—methodologically speaking—and they integrated and applied the new mathematics advanced by the Arabs (that is, algebra) to solve ancient problems.

Of course, it does not take long before the technical contents of this monograph become difficult to follow for those with no training in mathematics, especially when we arrive at the mathematics of the nineteenth century. But Pesic attempts to skate over these difficulties by including these technical developments in boxes that may be omitted by the reader, and Abel's original paper—in a completely new English translation—is remitted to an appendix.

There are other resources available to the reader. In the first place, in spite of its brevity, there is a concise and precise index to find specific topics. Moreover, there are informative notes that direct the reader to specialized sources. Unfortunately, these notes become useless, because they simply appear at the end of the book without any restricted system of numeration and reference. This was perhaps an editor's decision, one that is very unfortunate, because the user cannot guess which topics find their way into the notes. Quite possibly the reader may turn to the end of the book in search of the notes and not find the issues that he considers difficult.

The Sixteenth Century Journal **36**$_3$ (2005) 914-916