

Geometrical Landscapes: The Voyages of Discovery and the Transformation of Mathematical Practice. Amir R. Alexander. Stanford: Stanford University Press, 2002. 293 pp. \$65.00, ISBN 0-8047-3260-4.

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This penetrating and insightful book advances a thesis made original by the fact that we often approach the history of specific intellectual disciplines—especially in the sciences—isolated from the rest of human activities. Historians of science and mathematics usually pay more attention to politics, religion, philosophy, etc., but pay much less attention to issues of daily life. Most textbooks on the history of mathematics, for example, briefly discuss themes such as voyages, geography, nonwestern cultures, etc., as long as these arguments throw some light on the understanding and evolution of mathematical ideas and, implicitly, when innovative mathematical notions came to western culture. See, for example, George G. Joseph's *The Crest of the Peacock: The Non-European Roots of Mathematics* (London: Penguin, 1991).

In this brief, handsomely illustrated, and coherently structured monograph, Alexander attempts "to trace [the vision of mathematical study as an adventurous voyage of discovery] in the way mathematics was described and perceived, track its origins, and outline its implications for mathematical practice" (199–200). In fact, "the main thesis of this book [is] that the imagery of a mathematics of adventure and exploration went hand in hand with the emergence of infinitesimal methods" (200). Traditionally, historians of ideas are trained to analyze the tools available to those people struggling with specific problems, given their specific time and place, and to study the problems they were attempting to solve. Scholars are especially concerned with how, when, and where technical concepts were introduced, used, and interpreted. Alexander goes even further and pays close attention to the rhetoric of the texts as indicative of a general mental environment infusing a sense of adventure among early modern mathematicians as a direct consequence of their knowledge of the voyages of discovery. In short, early modern mathematicians regarded themselves as developing new mathematics and as real "explorers and discoverers."

According to Alexander, a key mathematical concept incorporating this pioneering sense of accomplishment lies in the analysis of diverse mathematical paradoxes that showed the strength and power of mathematical reasoning. Alexander argues that earlier mathematicians tried to avoid paradoxes (189), but such was not the case for early modern mathematicians such as Torricelli and, especially, Galileo. It is well known that in Galileo's *Two New Sciences*, the "first day" discussion between Salviati, Sagredo, and Simplicio focuses on some apparent paradoxes in number sequences. In particular, Salviati constructed an argument showing that square numbers $(1, 4, 9, 16, \dots, n^2, \dots)$ are as numerous as all numbers in general, which seems an absurd argument, and hence its paradoxical character (see Galileo's *Two New Sciences*, trans. Stillman Drake [Madison: University of Wisconsin Press, 1974], 40). Nevertheless, this "paradoxical character" seems to disappear when Salviati himself denied the possibility to conceive infinite numbers (Galileo, 41). If this were the case, then it is possible to conceive that there are not "paradoxical arguments" in Galileo's rhetoric, but indirect and informal *reductio ad absurdum* proofs, where Galileo deduces an absurdity from the contradiction of the proposition to be proven. But even if this were the case, Alexander's main argument would hold.

Finally, a strictly editorial comment. Placing the numerous and long notes at the end of the book makes the reading slow and discontinuous. Perhaps the publishers faced typesetting

difficulties when attempting to place the notes at the bottom of each page. But there were other possible alternatives. The strictly bibliographic notes could have been added, contained in brackets, directly into the text. In a second case, perhaps, it was also possible to include some of the long notes as part of the main text and keeping, at the bottom of the page, only those notes that added tangential information to the main argument. But, of course, these mundane and relative difficulties do not deprive the book of its enormous strengths.

