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REVIEWS

Edited by Catherine Goldstein and Paul R. Wolfson

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El nacimiento de la teoría de conjuntos, 1854–1908. By José Ferreirós Domínguez. Colección de Estudios, n° 39. Madrid (Ediciones de la Universidad Autónoma de Madrid), 1991. 394 pp.

Reviewed by Alejandro R. Garciadiego

Departamento de Matemáticas, Facultad de Ciencias, UNAM 04510 México D.F, México
E-mail: gardan@servidor.unam.mx

This is not just another book on the history of the theory of sets. Ferreirós has written a penetrating readable monograph. Extremely well acquainted with the primary and secondary literature, he openly acknowledges his intellectual debt to his colleagues (e.g., Hawkins, Mehrtens, and Scholz, among others). Moreover, Ferreirós does not restrict himself to an examination of the specific internal technical questions on which mathematicians base their research; he also attempts to link these questions with the external institutional context in which mathematicians develop and nurture their ideas. As a result, he has produced a broadly based history of the German origins (1854–1908) of this very popular branch of mathematics.

When considering the foundations of mathematics in the 1850s and the implications of the Göttingen and Berlin schools for mathematical research at that time, Ferreirós's analysis alerts the reader to possible differences between these two

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schools in terms of style, focus, and emphasis. This historical analysis allows the reader to understand the different intellectual backgrounds of Richard Dedekind and Georg Cantor while it outlines the historical influence and subsequent criticism of both approaches.

Furthermore, Ferreirós's methodological emphasis leads him to support a rather radical, but clear and well-argued, historical interpretation of the origins of set theory, focused on the role of Dedekind. He makes clear how, by the mid 1850s, Dedekind had already developed his abstract set-theoretic approach, which emphasized: (1) Dedekind's acceptance of the actual infinite; (2) his abstract focus on the concept of a group; and (3) the development of the concept of application. Ferreirós goes on to argue that it is Dedekind who chose the abstract approach and adopted purely existential concepts and proofs.

Ferreirós devotes Chapter VIII to the historical analysis of Cantor's concept of continuous and transfinite ordinals. Avoiding a polemical narrative, Ferreirós makes clear how Cantor structured his theory of transfinite numbers on the basis of the concept of cardinal number and ordinal type. He discusses how Dedekind influenced Cantor through his ideal theory, a point which is central to Ferreirós's main thesis; their correspondence is reviewed, as are their different points of view and their separation.

Chapter X, entitled "Philosophical Aspects in the History of the Theory of Sets," is one of the most interesting in the book. Here, once again, Ferreirós provides a broad and profound analysis of developments in this area. He discusses some of the common philosophical elements among Riemann, Cantor, and Dedekind. Leibniz, perhaps the most influential German philosopher of the 17th century, turns out to be a key figure. To some extent, Leibniz, according to Ferreirós, contributed to the intellectual acceptance of the concept of infinity. Similarly, Ferreirós argues, Leibniz possibly facilitated the acceptance of ideas associated with logicist premises.

When deliberating on the historical background of these events, Ferreirós pays close attention to the connotations of the language used at that time, in his attempts "to respect the complexities of the historical development" (p. 63). He carefully considers Riemann's use of the words *magnitud* (Grösse) and *variedad* (Mannigfaltigkeit) as synonyms and argues meticulously why his successors adopted the second one. But, in the epilogue of the account, in those events influenced by the work of Dedekind and Cantor, Ferreirós is not so careful. For example, when commenting on the origins of the set-theoretic "paradoxes," he treats the word *antinomia* as equivalent to *paradoja* (p. 351). The reader may be confused by Ferreirós's use of the term "antinomy," because he had previously discussed the influence of Kant's thought on German scholarship in the 19th century. Without any warning not to do so, it is logical to expect the reader to interpret "antinomy" in Kantian terms. Furthermore, Ferreirós employs this same term when discussing the mathematical and philosophical background of Russell's paradox. Although there are no explicit and precise definitions of the words "antinomy," "contradiction," and "paradox" in Russell's *The Principles of Mathematics* (1903), it is clear from Russell's writings that he did not intend these terms to be synonyms. Ferreirós has fallen into the

common trap of putting modern connotations into the mouths of historical figures, even though he was trying to avoid doing so.

The following changes should be incorporated into an English version, for which there is a real need. First, it would be desirable to avoid listing bibliographical references in footnotes. This practice disrupts the rhythm of the narrative of the text. According to standard writing style, Ferreirós should cite authors in the main body of the text, followed by the year of the publication in brackets. Second, it is imperative to stop indicating footnotes with numbers in parenthesis; they should be typed simply as superscript characters in order to avoid possible confusion with mathematical formulas (p. 179). Third, footnotes should begin with 1 (one) on every page, to avoid the use of high numbers near the end of each chapter. This would make the text easier to read. Fourth, and most important, it is essential to include a general thematic index at the end of the book. Like some other Spanish printing houses, *Ediciones UAM* decided to save money (and time) at the expense of one of the most indispensable tools of any book.

Finally, as we know, every book has its own history. No doubt, this monograph is the result of a long, well-planned, and well-designed process of research and writing. Perhaps, one day, José Ferreirós might tell us the story, as an anecdote to the discipline, of how a footnote—typed in English—found its way into the original Spanish edition (p. 72, note 30). There is a second footnote printed in German (p. 87, note 73), but perhaps, in this case, Ferreirós's intention was to provide us with the original text.

REFERENCES

Joseph Dauben, *Georg Cantor: His Mathematics and Philosophy of the Infinite*, Cambridge, MA.: Harvard Univ. Press, 1979.

Pierre Dugac, *Richard Dedekind et les fondements des mathématiques*. Paris: Vrin, 1976.

Alejandro R. Garciadiego, *Bertrand Russell and the Origins of the Set Theoretic Paradoxes*. Basel: Birkhäuser, 1992.

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