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SHOULD WE TRUST PRIMARY SOURCES? BERTRAND RUSSELL: A CASE STUDY*

ALEJANDRO R. GARCIADEGO**

1.

In spite of a much greater interest in methodological questions, the historiography of mathematics is much less well developed than that of other sciences. Of course, there are notable exceptions. Until recently, historians began to question the immutability and cumulative development of mathematics¹. Some other colleagues have noted that 'revolutions' do occur in mathematics².

Nevertheless, we could scan the pages of the leading research journals in this discipline and find that not many articles have been published on historiographical questions in the last twenty years. In addition, a recent publication serving

* This is a slightly revised version of the conference presented at the XVIII International Congress on the History of Science. I made some modifications as a result of constructive criticism and insightful suggestions offered by some colleagues present at the meeting. I tried to keep, as far as possible, the informal character of the presentation of the lecture. All the footnotes have been added to the printed version. The reader interested in a more rigorous version of the ideas related to Russell's writing of *The Principles of Mathematics* should consult: Alejandro R. GarciaDiego, *Bertrand Russell and the Origins of the Set Theoretic Paradoxes*, Birkhäuser (forthcoming). Finally, but most importantly, I would like to express my deepest gratitude to S.S. Demidov, M. Folkerts and J. Dauben for their kind invitation to participate in the conference. I would like to make public also my thankfulness to N. White for his valuable comments on a previous typescript.

¹ Michael Crowe, "Ten Misconceptions in the History of Mathematics", in William Aspray, Philip Kitcher (eds), *History and Philosophy of Modern Mathematics* (Minneapolis: University of Minnesota Press, 1985), 260-277. For an earlier example, see: Charles S. Fisher, "The Death of a Mathematical Theory: a Study in the Sociology of Knowledge", *Archive for History of Exact Sciences*, 3 (1966), 137-159.

² See Joseph W. Dauben, "Conceptual Revolutions and the History of Mathematics: Two Studies in the Growth of Knowledge", in Everett Mendelsohn (ed.), *Transformation and Tradition in the Sciences* (Cambridge, Mass.: Cambridge Univ. Press, 1984), 81-103. See also, among others: Herbert Mehlert, "T. S. Kuhn's Theories and Mathematics: a Discussion Paper on the 'New Historiography' of Mathematics", *Historia mathematica*, 3 (1976), 287-320.

** Departamento de Matemáticas
Facultad de Ciencias
Universidad Nacional Autónoma de México
04510 México, D.F. México

as an introductory guide into the history of mathematics (listing the most important and sometimes the only existing sources available) lacks a historiographical section. There are a few exceptions; these articles are listed under the heading 'sociology of mathematics' or 'philosophy of mathematics'³.

Some historians question whether historiographic discussions shed any new light into the history of mathematics itself. Similarly, they may question whether research on the 'scientific method' enriches our understanding of nature or contributes to the development of the natural sciences. Therefore, they claim that these studies are not worth pursuing. This is a reasonable position.

Scanning the pages of those same leading journals once again, it is readily noticeable that many historians of mathematics do not list one single item belonging to the secondary literature of the theme in question in the references or bibliography. Regardless of whether the author is investigating the metamorphosis of an idea, or developing the biography of a person, the historian turns to relevant articles of original research, autobiographical notes, photographs, films, personal correspondence, unpublished material and the like. We judge the material analytically and implicitly trust the veracity of its contents. Perhaps, we also tacitly suppose that the authors of these materials expected us to read them literally. In short, we attempt to show our comprehension of the topic by solely relying on primary sources. But, is it possible to do so? And, is this practice healthy for our discipline? We could even ask ourselves, why do we write history if we do not read it?

I will attempt, first, to show that the indiscriminate and uncritical usage of primary sources could result in misleading or erroneous reconstructions. In particular, I will sketch the historical consequences of two different (but plausible) reconstructions of Russell's writing of *The Principles of Mathematics* (published in May 1903). Secondly, we have to assume that, in some cases, historical figures have deliberately left misleading clues. Using other primary sources, I will briefly discuss how Russell misled the readers (especially reviewers) of the second edition of the same book.

2.

From Russell's autobiographies it is possible to infer immediately that Russell began to write the first version of his final draft of *The Principles of Mathematics* in October 1900 and finished it, as he claims, on the last day of the nineteenth century. He also claims to have entirely finished the final ver-

³ Joseph W. Dauben (ed.), *The History of Mathematics from Antiquity to the Present. A Selective Bibliography*, New York, Garland, 1985.

sion on May 23, 1902.⁴ I may add that there are surviving records (kept at The Bertrand Russell Archives) from those specific days of 1900 and 1902, respectively, that support his claims.⁵ Moreover, in the printed preface of the book, originally written in December 1902, when the memories were still fresh, Russell narrates the intellectual path he followed especially with respect to the philosophical and mathematical influences that affected him during its composition.⁶ This preface also seems to support Russell's later recollections.

So, nothing seems to need clarification! We have inherited Russell's recollections and they are based on sources written at the time of the event. Moreover, Russell claims that the central parts of the book (i.e. parts III to VI) were published almost exactly as he wrote them during the autumn of 1900. Some of the folios belonging to these parts were dated, in Russell's handwriting, during the time between November and December 1900, supporting even more strongly Russell's own memories.⁷ So far, so good!

From these and other of his recollections we could infer, among others, the following statements:

1. Russell began to write the first attempt of his 'big book', as he used to call it, in October and finished it on 31st December 1900. If this is the case, then Russell had arrived at some sort of his 'logician thesis', the central assertion of the book, by October 1900.

2. He felt it necessary to rewrite some parts of the manuscript as a consequence of the discovery of the set theoretic paradoxes and, therefore, wrote a new version of Parts I and II in May and June 1901. If this is the case then Russell understood precisely the importance of the 'paradoxes' by this time.

3. Once again, in May 1902, he revised Parts I, II and VII and composed a third version of the book.

3.

Nevertheless, there are other primary sources that could portray a slightly different picture. For example, when Russell described to Philip Jourdain (a

⁴ Bertrand Russell, *The Autobiography of Bertrand Russell, 1872-1914* (London: George Allen and Unwin, 1967), 145; *id.*, *My Philosophical Development* (London: Unwin books, 1975 [originally published in 1959]), 56.

⁵ See letters from Russell to Helen Flexner of Dec. 31, 1900; and to his wife Alys Russell on May 24, 1902, The Bertrand Russell Archives, McMaster University, Hamilton, Ontario, Canada.

⁶ Bertrand Russell, *The Principles of Mathematics*, (London: Cambridge Univ. Press, 1903), xvi-xviii.

⁷ The manuscript used by the printers of Cambridge University Press survives at The Bertrand Russell Archives.

professional mathematician, logician and historian) how he came to write *The Principles of Mathematics*, he stated that during the last three months of 1900 he wrote parts III to VI. He, then, added in parenthesis: "(Part VII is largely earlier, Parts I and II wholly later, May 1902)"⁸. Here, Russell introduces a new element to the discussion. What does he mean when he says that Part VII was written 'largely earlier'? Does he imply that he developed this part of the book even before he sat down to compose the first attempt of his now famous treatise? Other primary sources also contradict Russell's assertion that he finished this attempt of *The Principles of Mathematics* by 31st December 1900. According to the personal diary of Mrs Russell (née Alys Pearshall Smith), her husband had only completed two thirds of his book on the foundations of mathematics by that date. Thirdly, it appears that when Russell wrote the original folios of Part V, he ignored the contents of earlier sections of the book (e.g. Parts I and II)⁹. This fact might imply that Russell did not write his book in sequential order, running from Chapter I onwards.

This new information provides a contradictory account of the events surrounding the writing of *The Principles of Mathematics*:

1. Russell began the first version of the final manuscript of his 'big book' in October 1900 but did not finish it until January 1903. In this case, Russell began by writing Part III. He did not possess knowledge of the "logician thesis" when he started the book. This is perhaps the most important consequence of this second interpretation.

2. Russell wrote the first version of Parts I and II in May and June 1901, without accounting for the existence of the contradictions. Nevertheless, the manuscript was still incomplete: Part VII was not incorporated until May 1902.

3. In May 1902, Russell was forced to rewrite Parts I and II as a consequence of the discovery of the set theoretic paradoxes and to incorporate Part VII from a previous manuscript simply entitled "Principles of Mathematics" finished in June 1900.

The second account of the events surrounding the writing of *The Principles of Mathematics* makes it possible to explain how Russell developed the "logician thesis" after he had started his manuscript in October 1900. All references to this thesis in Parts III to VI seem to have been added later. It is also possible to explain the influence Cantor exerted on him, especially once Russell read

⁸ Letter from Russell to Jourdain, dated 15 April 1910, quoted in Ivor Grattan-Guinness, *Dear Russell – Dear Jourdain* (London: Duckworth, 1977), 133.

⁹ See Bertrand Russell, "The Principles of Mathematics", part V, chapter III, folio 40, The Bertrand Russell Archives.

Cantor's papers of 1895 and 1897;¹⁰ and, finally, it is possible to analyze how he discovered the set theoretic 'paradoxes' while he was composing his treatise.

4.

One may argue that it is a common practice to find contradictory quotations in the works of most scientists and philosophers. Knowledge is a living organism, and ideas transform and evolve. It is not unusual to find intellectuals accepting ideas they had previously rejected; or vice versa, rejecting concepts that they previously accepted. Nor is it unusual to find contradictory information in the historical records at our disposal. Furthermore, we are all well aware of the dangers of relying on biographical or, even worse, on autobiographical sources. But is it possible, as I suggested in the title, to encounter *misleading* clues left for us in the primary sources? I will provide an example using Russell and his book.

In 1938, Russell published the second edition of *The Principles of Mathematics*. By this time, he had already almost abandoned his original interests in the philosophy of mathematics and was concerned mainly with pedagogical questions; not only that he was not holding an academic position but was surviving by publishing books addressed to the general public. It is reasonable to believe that he was not familiar with the latest developments in mathematical logic and general set theory. At this time, he knew he was only superficially acquainted with some of the most important recent contributions to the field.¹¹

When this second edition of *The Principles of Mathematics* appeared, Russell added a new introduction. This new introduction is one of the few places in which Russell commented on the ideas of Hilbert and Brouwer on the foundations of mathematics. It is, therefore, of great importance to philosophers, historians and mathematicians. But this topic is tangential to our purpose today. The essential issue is Russell's observation on the main thesis of the sec-

¹⁰ Cantor's powerful influence on Russell was originally obscured by Russell's own comments (see: Bertrand Russell, *The Autobiography ...*, cit., 144-145, 217). Other assessments have been published recently, see: Ivor Grattan-Guinness, "Georg Cantor's Influence on Bertrand Russell", *History and Philosophy of Logic*, 1 (1990), 61-93; and Alejandro R. Garciaadiego, "L'influence de Georg Cantor sur Bertrand Russell", *Cahiers du séminaire d'histoire des mathématiques de Toulouse*, 8 (1986), 1-12.

¹¹ In the first edition of Schilpp's volume on Russell for the 'Library of the Living Philosophers', Russell refused to comment on Gödel's essay treating the development of Russell's mathematical logic, arguing that this particular piece of work arrived too late and that he was incapable of judging Gödel's opinions. Some years later, in a second edition containing an "addendum to [Russell's] reply of criticisms", Russell did not care to comment.

ond edition of his own book: "The fundamental thesis of [this book], that mathematics and logic are identical, is one which I have never since seen any reason to modify"¹².

This new edition of *The Principles of Mathematics* was reviewed in popular newspapers as well as professional journals in mathematics, philosophy and even history of science¹³. All of the commentators, perhaps influenced by Russell, seem to agree at least on one vital point. The central thesis of the book, that mathematics and logic are identical, was still maintained by Russell.

But, was this the original thesis of the book? Even if Russell says so? If we carefully read Russell's preface to the first edition and all the added notes in the manuscript concerning the "logician thesis" we might come to a different conclusion. In his book, Russell claims that pure mathematics is deducible from a very small number of fundamental logical principles and, therefore, that mathematics is a branch of logic. But, it is not identical¹⁴. The difference might be very subtle, but clear, at least, for Russell.

I believe that Russell was well aware of this subtle distinction. Philosophers, and especially mathematicians, had overwhelmingly opposed this viewpoint for the last thirty-five years. In particular, Gödel's 1931 result directly challenged any attempt to completely axiomatize arithmetic following the logicist principles, as stated in *Principia mathematica*, and therefore in *The Principles of Mathematics*.

Russell knew that his original thesis was untenable. Ten years before, when questioned by Jourdain, Russell informed him that in order to publish a second edition of *The Principles* ... radical changes were necessary¹⁵. If this was the case, then there was no justification to print a second edition of *The Prin-*

¹² Bertrand Russell, *The Principles of Mathematics* (New York: W. W. Norton & Company, 1938), v.

¹³ See, among others: Eda Walton, *The Nation*, 146 (1938), 733-734; T. G., *Nature*, 142 (1938), 662-663; Albert E. Meder, *Scripta mathematica*, 7 (1940), 138-141; H. G. F., *The Mathematical Gazette*, 22 (1938), 300-301; P. R., *Rivista di filosofia*, 43 (1951), 457-458; Susanne K. Langer, *The Journal of Symbolic Logic*, 3 (1938), 156-157; L. Susan Stebbing, *Philosophy*, 13 (1938), 481-483; H. T. Davis, *Iris*, 30 (1939), 298-302.

¹⁴ It is surprising that Russell did support this 'identity' in an earlier article (see Bertrand Russell, "Recent Work on the Foundations of Mathematics", *The International Monthly*, 4 [1901], 83-101, especially pages 83, 85), but not in *The Principles of Mathematics*.

¹⁵ "In revising 'The Principles' is there any hope that you would reprint with added notes in square brackets showing how your views have been modified?" (Letter: Jourdain to Russell, 6 September 1917). Russell replied: "I do not think that in revising the 'Principles' I can adopt the plan you suggest. The whole of the first part would need to be entirely and completely re-written; so would the third part; and the second part would need very tremendous alterations" (Letter: Russell to Jourdain, 11 September 1917). Both letters are quoted in Ivor Grattan-Guinness, *Dear Russell ...*, cit., 142-143.

inciples of Mathematics making the same assertion. At least a slight modification was required, softening the original claim that mathematics was derived from logic. But, the audience should keep in mind that Russell's new statement of his thesis is not supported by the rest of his text.

5.

Historians agree that every source – whether a primary or a secondary one – has to be read with the same critical and sceptical eyes. I would like to rephrase the original title of this paper as follows: Are primary sources a necessary and a sufficient condition for reconstructing the past? Perhaps, this is the case. But, I believe that a negative answer, at least to the question of sufficiency, would enrich the possibilities and methods of the study of the history of mathematics.