90c:01051 01A65 01A72 03F99 46R05

Mandelkern, Mark (1-NMS)

Brouwerian counterexamples.

Math. Mag. **62** (1989), *no.* 1, 3–27.

Brouwerian counterexamples are "demonstrations showing that their truth would imply solutions to problems for which, in fact, no solutions were known" (p. 3). Following this characterization, the author explains the technical details associated with these counterexamples. (For those wishing to find a clear and brief explanation of some of the basic principles used as a foundation for this conception, see another paper by the author [same journal **58** (1985), no. 5, 272–280; MR 87b:03001].)

The most striking feature for the examples used by the author is their meaning and importance for "classical mathematics". For example, in order to show the nonconstructive nature (i.e., lack of numerical meaning) of the least upper bound principle, the author demonstrates that if this was not the case then this principle "would provide a finite method leading either to a proof of Fermat's last 'theorem' or to an explicit counterexample", in the traditional meaning of the word (p. 4). A second example shows that the principle of trichotomy of real numbers implies a solution to the perfect number problem. A third example, associated with the discontinuous function principle, presents a solution to Goldbach's conjecture.

The reader should not get the impression of finding just pathological cases. In a coherent and comprehensible way, the author shows how it is possible for "constructive workers [to] pick up the pieces and remold them into a number of different constructively valid theorems, each displaying a different aspect of the situation" (p. 18).

Finally, the article—ideal to introduce students and nonspecialists to this field—contains a well-documented bibliography of some recent constructive developments.

Alejandro R. Garciadiego (Mexico)