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Goldfarb, Warren (1-HRV-Q)

Poincaré against the Logicists.

History and philosophy of modern mathematics (*Minneapolis, MN, 1985*), 61–81, *Minnesota Stud. Philos. Sci., XI, Univ. Minnesota Press, Minneapolis, MN*, 1988.

Aspray and Kitcher have edited the book titled *History and philosophy of modern mathematics* in which this paper appears. The book is the result of a conference held at the University of Minnesota (Minneapolis, 17–19 May 1985) and, its "purpose is to bring together a number of different perspectives on modern mathematics, with the aim of understanding how the work of historians, philosophers and mathematicians can be integrated in an interdisciplinary study of mathematics" (p. vii). The present essay is part of the first section entitled: "Logic and the foundations of mathematics".

The title of the author's essay might suggest a historical treatment of the topic. Perhaps the reader expects a chronological reconstruction of how Poincaré's ideas evolved during his debate with the Logicists. But this is not the case. Instead, the reader finds a profound and coherent philosophical analysis of Poincaré's conception of two basic principles and their implications concerning a criticism of the Logicist program: a petitio argument and the vicious circle principle. At the end, the author's convincing arguments show that Poincaré's strictures were supported by a distinctive conceptualization of the foundations of mathematics (p. 79), a conceptualization that was irreconcilable with those efforts (especially Frege's) to eradicate psychology from mathematics.

The author's title, on the other hand, is not exclusive to those who followed Frege's and Russell's logicism, or some sort of it. In fact, he discusses some of those supporting the "new logic", and the reasoning also involves Zermelo's points of view. Other names are also mentioned (e.g., Cantor, Peano and Hilbert), but incidently debated.

The author introduces into his analysis some comments on the set-theoretic paradoxes, in particular the one attributed to Jules Richard, where Poincaré presented the vicious circle principle. The paradoxes in themselves—or antinomies as Poincaré called them, perhaps following a Kantian tradition—were elements confirming that the new logic had no development at all during the previous quarter century. It was during the polemic with Couturat that Poincaré attributed a paradox to Burali-Forti, a paradox the latter failed to recognize at first. But, the reviewer also claims, contrary to the author's assertion (p. 71) that it was a result of this same debate (and as a consequence of some other publications of Jourdain), that Russell came to realize the importance of the Burali-Forti paradox—an argument he attempted to avoid by showing that the set of all ordinal numbers was not well-ordered [see B. Russell, *Principles of mathematics*, see p. 323, Cambridge Univ. Press, Cambridge, 1903; Jbuch 34, 62]; G. H. Moore and the reviewer [Historia Math. 8 (1981), no. 3, 319–350; MR 83d:01037].

This same analysis explains Poincaré's refusal to acknowledge the notion of a fixed range of quantifiers and his denial of the existence of the actual infinite—a position that will force him to abandon Cantor's Mengenlehre. But, perhaps more important, the author is even capable of generating a global picture of Poincaré's views on mathematics. And this is a picture difficult to obtain from Poincaré's popular books and articles.

Finally, the Workshop on the Evolution of Modern Mathematics (organized by Garrett Birkhoff in 1974) had served, according to the editors, as a model to organize this Conference on History and Philosophy of Modern Mathematics. Although it is impossible, at the moment, to quantify the impact of the first Workshop, most historians would agree it played an important role in the recent professionalization of their discipline. Perhaps this latest conference, as is clear through the works of the author and Kitcher (among others), might generate an impulse to professionalize the philosophy of mathematics as well.

(For the entire collection see MR 89b:00023.)

Alejandro R. Garciadiego (Mexico)

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