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The conveyability of intuitionism, an essay on mathematical cognition.

J. Philos. Logic **17** (1988), *no.* 2, 133–156.

This well-written, knowledgeable and balanced article attempts to prove that an intuitionist understanding of number and of logical constants "must be inextricably linked" and, furthermore, that answers to these questions "should undermine an influential argument for intuitionistic over classical mathematics by raising doubts about supporting views on the nature of mathematical cognition and its genesis" (page 133).

In order to carry out his task, the author unveils and explores possible arguments used by intuitionists (represented by Dummett's position) in their debates concerning these questions with Platonists (Gödel (?)) and actualists (Bernays), and arrives at the conclusion that, on the rebound, all intuitionist characterizations of natural numbers exhibit, sooner or later, some kind of circularity.

Philosophically speaking, his arguments show coherence and completeness—just when the reader may be afraid of losing an argument, an Ariadne's thread is always dropped. Furthermore, the author's penetrating philosophical analysis discloses further philosophical—as well as historical—questions that might be worth pursuing. In the first place, one feels strongly motivated to apply the author's analysis to other contemporary schools of mathematical thought (e.g., holism and quasi-empirical realism, among others).

On the other hand, historically speaking, the analysis leads him to wonder why "no one before the 17th century appears even to have formulated this basic feature (induction) of mathematical intuition" (page 140). He mentions Brouwer's influence, but does not elaborate on possible causes or origins. This is most important, since some historians have argued that Brouwer's dissatisfactions with the philosophical foundations of mathematics at the beginning of the present century—and his reasons for presenting alternative views—were not mathematical in nature [see W. P. van Stigt, Historia Math. **6** (1979), no. 4, 385–404, see p. 388; MR 80k:01048]. Following the author's comments on Brouwer and his relation with Poincaré, the author asserts "that it is hard to understand how Brouwer could have endorsed Poincaré's suggestion of mutual unintelligibility". Let us keep in mind that Russell had advanced similar views when he defined mathematics as "the subject in which we never know what we are talking about, nor whether what we are saying is true" [B. Russell, Internat. Monthly **4** (1901), 83–101, see p. 84; reprinted in *The world of mathematics, Vol. III*, 1576–1590, Touchstone, New York, 1956; per revr.]. And then we face a deeper question: is it possible that Russell also endorsed Poincaré's views and, therefore, also Brouwer's?

After reading this intelligent paper, the peruser will be convinced that the philosophy of mathematics, as a branch of mathematics, is alive and well and that it is expressible in direct and simple terms.

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